**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Sol:B.0.2676

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0)

= Pr(Z ≤ 0.625)

=73.4%

Probability of given

= 100-73.4 = 26.6% or 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Solution:**

Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %

=> People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400

Z score for 38  = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is F**ALSE**

Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Sol:** CLT states that ,Let X1, . . . , Xn be independent r.v.s with mean µ and variance σ2 , from ANY distribution. For example, Xi ∼ Binomial(n, p) for each i, so µ = np and σ 2 = np(1 − p).

Sn = X1 + X2 + . . . + Xn → Normal(nµ, n σ2 ) as n → ∞.

So, X1 + X2=Normal(2µ, 2 σ2 )

Also,X1+X1=2X1=Normal(2µ, 2 σ2 )

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Sol**: Given:  p(a<x<b) = 0.99 ,mean =100,standardDeviation = 20

To Find:

Z=(X- μ) / σ

Probability 0.005 the Z Value = -2.57

X=Z \* σ + μ

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Sol: μ ± 2σ

= 540 ± 2\*225

= 540 ± 450

= (540 - 450, 540 + 450)

=RANGE OF (90,990)

1. Specify the 5th percentile of profit (in Rupees) for the company

Sol: = μ - 1.5σ

=540 - (1.5 \* 225)

=202.5(In Millions)

1. Which of the two divisions has a larger probability of making a loss in a given year?

Sol: division1= Z score for a profit of zero: Z=(X-µ)/ *s.d*

*=*  (0-5)/3

= -1.66=0.0485

division2= Z score for a profit of zero: Z=(X-µ)/ *s.d*

=(0-7)/4

= -1.75= 0.0401

Division 1 has a larger probability of making a loss in a given year